Extended Mean Cordial Graphs of Snakes

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Extended Mean Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1,2} such that each edge uv is assigned the label $(\lceil f(u)+f(v)\rceil)/2$ where ⌈ x ⌉ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that Path related graphs Tn, Q(n), DQ(n), TQ(n) are Extended Mean Cordial Graphs.

Index Terms – Extended Mean Cordial Graph, Extended Mean Cordial Labeling, 2000 Mathematics Subject classification 05C78.

1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that Path related graphs Tn, Q(n), DQ(n), TQ(n) are Extended mean Cordial Graphs. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Extended Mean Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1,2} such that each edge uv is assigned the label $(\lceil f(u)+f(v)\rceil)/2$where ⌈ x ⌉ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that Path related graphs Tn, Q(n), DQ(n), TQ(n) are Extended Mean Cordial Graphs.

Definition: 2.1

A triangular snake is obtained from the path $(v_1,v_2,\ldots\ldots,v_n)$ by replacing every edge by a triangle C_3. It is denoted by T_{n-1}.

Definition: 2.2

A quadrilateral snake Q(n) is obtained from a path $(u_1, u_2,\ldots\ldots, u_n)$ by joining u_i , u_{i+1} to new the vertices v_i and w_i respectively and joining the vertices u_i , v_i and u_i , w_i also joining the vertices u_{i+1} , v_i and u_{i+1} , w_i respectively. (i.e) every edge of the path is replaced by a cycle C_4.

Definition: 2.3

It is a graph obtained from a path P_n, by replacing every edge of a path by two quadrilateral s and it's denoted by DQ (n).

Definition: 2.4

It is a graph obtained from a path P_n , by replacing every edge of a path by three quadrilaterals and its denoted by TQ(n).

3. MAIN RESULT

Theorem 3.1

Triangular snake T(n) is a Extended Mean Cordial Graph.

Proof:

Let $V[T(n)] = \{ (u_i) : 1\le i\le n-1, (v_i) : 1\le i\le n\}$

Let $E[T(n)] = \{ [(v_i v_{i+1}) : 1\le i\le n-1]\cup[(u_i v_i) : 1\le i\le n-1]\cup [(u_i v_{i+1}) : 1\le i\le n-1]\}$

Define f: V(G)→{0,1,2} by

$$f(u_i) = \begin{cases} 2 \text{ if } i \equiv 1 \bmod 2 \\ 1 \text{ if } i \equiv 0 \bmod 2 \end{cases},\quad 1\le i\le n-1$$

$$f(v_i) = \begin{cases} 1 \text{ if } i \equiv 1 \bmod 2 \\ 0 \text{ if } i \equiv 0 \bmod 2 \end{cases},\quad 1\le i\le n$$

The induced edge labeling are

$f^*(v_i v_{i+1}) = 0,\quad 1\le i\le n-1$

$$f^*(u_i v_i) = \begin{cases} 1 \text{ if } i \equiv 1 \bmod 2 \\ 0 \text{ if } i \equiv 0 \bmod 2 \end{cases},\quad 1\le i\le n-1$$

$f^*(u_i v_{i+1}) =1,\ 1\le i\le n-1$

Here, $e_f(0) = e_f(1)$ when n is odd

$e_f(1) = e_f(0) +1$,n is even

Hence T_n is Satisfies the condition

| ef(0) – ef(1) | ≤ 1

Therefore, T_n is an Extended Mean Cordial Graph.

For example, T_n is an Extended Mean Cordial Graph as shown in the figure, 3.2

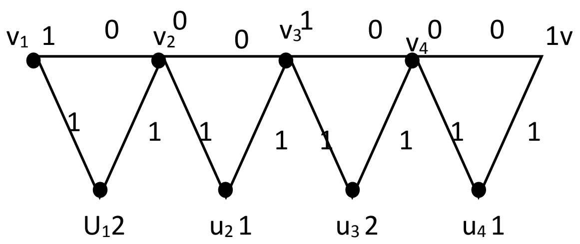

Figure 3.2

Theorem: 3.3

Quadrilateral Snake Q (n) is an Extended Mean Cordial Graph

Proof:

Let $V[Q(n)] = \{ (v_i): 1\leq i\leq n, (u_{i1},u_{i2}) : 1\leq i\leq n-1\}$

Let $E[Q(n)] = \{[(v_i v_{i+1})\cup(v_i u_{i1})\cup(v_{i+1} u_{i2}) :$

$1\leq i\leq n-1]\cup [(u_{i1} u_{i2}): 1\leq i\leq n-1]\}$

Define $f:V(Q(n))\rightarrow\{0,1,2\}$

$f(v_i) = \begin{cases} 0 \; if \; i \equiv 0 \; mod \; 2 \\ 1 \; if \; i \equiv 1 \; mod \; 2 \end{cases}, 1\leq i\leq n$

$f(u_{i1}) = 2$, $1\leq i\leq n-1$

$f(u_{i2}) = 0$, $1\leq i\leq n-1$

The induced edge labeling are

$f^*(v_i v_{i+1}) = 0$, $1\leq i\leq n-1$

$f^*(v_i u_{i1}) = 1$, $1\leq i\leq n-1$

$f^*(v_{i+1} u_{i2}) = 0$, $1\leq i\leq n-1$

$f^*(u_{i1} u_{i2}) = 1$, $1\leq i\leq n-1$

Here, $|e_f(0)-e_f(1)| \leq 1$

Hence Q (n) is Satisfies the condition| ef(0) – ef(1) | ≤ 1

Therefore, Q(n) is an Extended Mean Cordial Graph.

For example, Q(4) is an Extended Mean Cordial Graph as shown in the figure 3.4

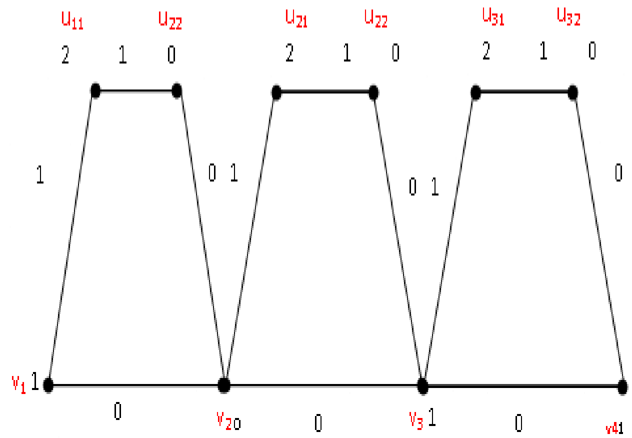

Figure 3.4

Theorem: 3.5

TQ (n) is an Extended Mean Cordial graph.

Proof:

Let TQ(n)= (V,E)

Let G be a TQ(n).

Let $V(TQ(n)) = \{[w_i: 1\leq i\leq n+1]\cup$

$[u_{i1}, u_{i2}, v_{i1}, v_{i3}, x_{i1}, x_{i2}: 1\leq i\leq n-1]\}$

Let $E(TQ(n)) = \{[(w_i: 1\leq i\leq n-1)\cup$

$[(u_{i1}u_{i2})\cup(v_{i1}v_{i2})\cup(x_{i1}x_{i2}): 1\leq i\leq n-1]\cup$
$[(w_iu_{i1})\cup(w_iv_{i1})\cup(w_ix_{i1}): 1\leq i\leq n]\cup[(w_{i+1}u_{i2})$

$\cup(w_{i+1}v_{i2})\cup(w_{i+1}x_{i2}): 1\leq i\leq n-1]\}$

Define $f:V(G)\rightarrow\{0,1,2\}$

$f(w_i) = \begin{cases} 1 \; i \equiv 0 mod \; 2 \\ 0 \; i \equiv 1 mod \; 2 \end{cases}, 1\leq i\leq n$

$f(u_{i1}) = 0$, $1\leq i\leq n-1$

$f(u_{i2}) = 2$, $1\leq i\leq n-1$

$f(v_{i1}) = 1$, $1\leq i\leq n-1$

$f(x_{i1}) = \begin{cases} 0 \; i \equiv 0 \; mod \; 2 \\ 1 \; i \equiv 1 \; mod \; 2 \end{cases}, 1\leq i\leq n-1$

$f(x_{i2}) = 1$, $1\leq i\leq n-1$

The induced edge labeling are,

$f^*(w_iw_{i+1}) = 0$, $1\leq i\leq n-1$

$f^*(v_{i1}v_{i2}) = \begin{cases} 0 & i \equiv 1 mod\ 2 \\ 1 & i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n-1$

$f^*(u_{i1}u_{i2}) = 1$, $1 \leq i \leq n-1$

$f^*(x_{i1}x_{i2}) = \begin{cases} 0 & i \equiv 0 mod\ 2 \\ 1 & i \equiv 1 mod\ 2 \end{cases}, 1 \leq i \leq n-1$

$f^*(w_i u_{i1}) = 0$, $1 \leq i \leq n-1$

$f^*(w_i v_{i1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 0 & i \equiv 0\ mod\ 2 \end{cases}, 1 \leq i \leq n-1$

$f^*(w_i x_{i1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 0 & i \equiv 0\ mod\ 2 \end{cases}, 1 \leq i \leq n-1$

$f^*(w_{i+1}u_{i2}) = 1$, $1 \leq i \leq n-1$

$f^*(w_{i+1}v_{i2}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 0 & i \equiv 0\ mod\ 2 \end{cases}, 1 \leq i \leq n-1$

$f^*(w_{i+1}x_{i2}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 0 & i \equiv 0\ mod\ 2 \end{cases}, 1 \leq i \leq n-1$

Here ef(0) =ef(1),

Hence TQ (n) is Satisfies the condition

| ef(0) – ef(1) | ≤ 1

Therefore, TQ (n) is an Extended Mean Cordial Graph.

For example, TQ (n) is an Extended Mean Cordial Graph as shown in the figure 3.6

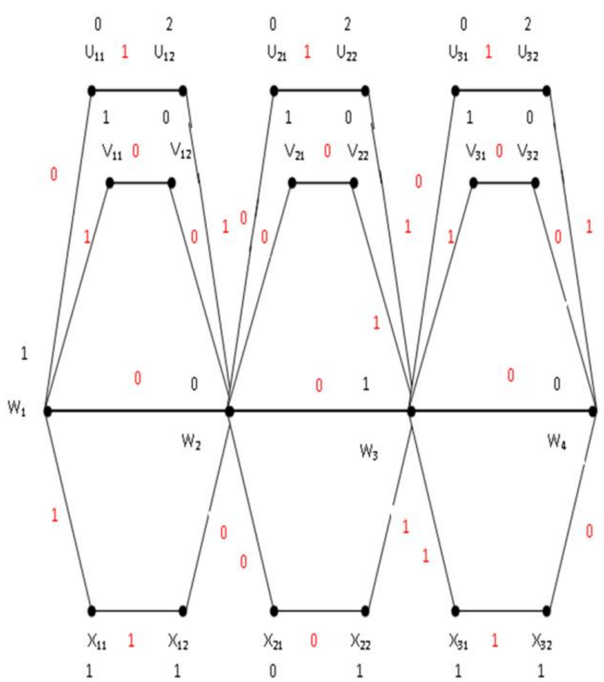

Figure 3.6

Theorem:3.7

Let D Q(n) is a Extended mean cordial graph.

Proof:

Let DQ(n) = (V,E)

Let DQ(n) be a Extended Mean Cordial Graph.

Let V(DQ(n)) ={ [v_i :1≤i≤n]U[u_{i1}, u_{12}, w_{i1}, w_{i2}: 1≤i≤n]}

Let E(DQ(n)) = {[($v_i v_{i+1}$) U(v_iu_{i1})U(v_iw_{i1})U($v_{i+1} w_{i2}$)U($v_{i+1} u_{i2}$) U($u_{i1}u_{i2}$)U($w_{i1}w_{i2}$) : 1≤i≤n]}

Define : f:V(DQ(n))→{0,1,2}

The vertex labeling are

$f(v_i) = \begin{cases} 1 & i \equiv 1 mod\ 2 \\ 0 & i \equiv 0 mod\ 2 \end{cases}$, 1≤ i≤n

$f(u_{i1}) = 2$,1≤i≤n

$f(u_{i2}) = 0$, 1≤i≤n

$f(w_{i1}) = 2$,1≤i≤n

$f(w_{i2}) = \begin{cases} o & i \equiv 1 mod 2 \\ 1 & i \equiv 0 mod 2 \end{cases}$, 1≤i≤n

The induced edge labeling are,

$f^*(v_iv_{i+1}) = 0$, 1≤i≤n-1

$f^*(u_{i1}u_{i2}) = 1$, 1≤i≤n-1

$f^*(w_{i1}w_{i2}) = \begin{cases} 0 & i \equiv 1\ mod\ 2 \\ 1 & i \equiv 0\ mod\ 2 \end{cases}$, 1≤i≤n-1

$f^*(v_iu_{i1}) = 1$, 1≤i≤n-1

$f^*(v_{i+1}u_{i2}) = 0$, 1≤i≤n-1

$f^*(v_iw_{i1}) = \begin{cases} 1 & i \equiv 1\ mod\ 2 \\ 0 & i \equiv 0\ mod\ 2 \end{cases}$, 1≤i≤n-1

$f^*(v_{i+1}w_{i2}) = \begin{cases} 0 & i \equiv 1\ mod\ 2 \\ 1 & i \equiv 0\ mod\ 2 \end{cases}$, 1≤i≤n

Here, ef(1) = ef(0) +1

Hence DQ(n) is Satisfies the condition| ef(0) – ef(1) | ≤ 1

Therefore, DQ(n) is a Extended Mean Cordial Graph.

For example, DQ(n) is a Extended Mean Cordial Graph as shown in the figure 3.8

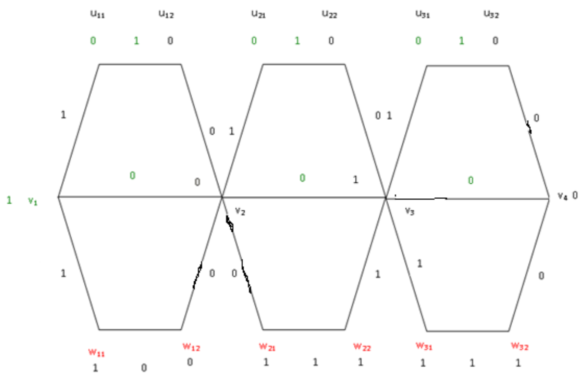

Figure 3.8

4. CONCLUSION

Mean cordial graph has some applications in digital theory. It is an attempt to extend the domain of vertex labeling and identify some of the graphs satisfying the extended graph labeling. This will result in wider applications.

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